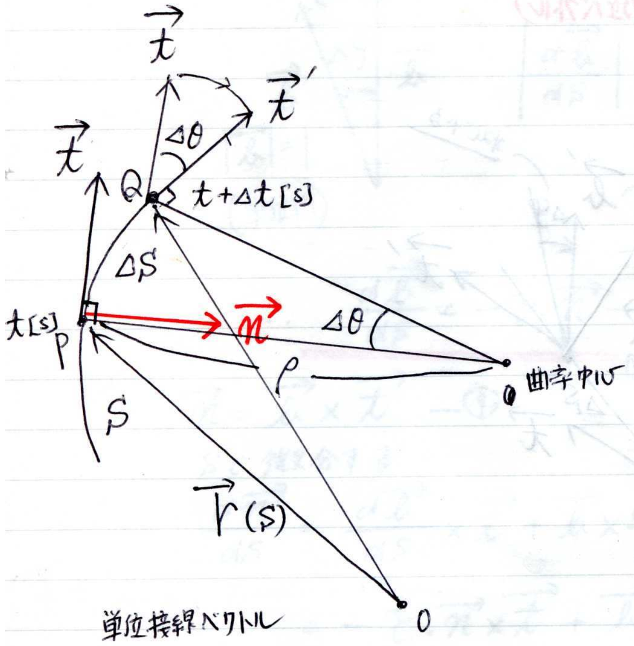
[空間曲線]



$$S = \int_{t_0}^{t} \left| \frac{d\vec{r}}{dt} \right| dt$$

$$\therefore \frac{dS}{dt} = \left| \frac{d\vec{r}}{dt} \right| > 0$$

単位接線ベクトル

$$\vec{t} = \frac{d\vec{r}(s)}{dS} \quad \text{—①} \qquad \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dS}\frac{dS}{dt} \quad \text{—②}$$

②より

$$\vec{t} = \frac{d\vec{r}}{dS} = \frac{\frac{d\vec{r}}{dt}}{\frac{dS}{dt}} = \frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|} \quad \text{—③}$$ 接線ベクトル

$|\vec{t}| = 1$(一定) なので Sについて 微分すると

$$\frac{d\vec{t}^2}{dS} = 2\frac{d\vec{t}}{dS}\cdot\vec{t} = 0 \quad \text{—④} \quad \therefore \frac{d\vec{t}}{dS} \perp \vec{t} \quad \text{—⑤}$$

$$\frac{d^2\vec{r}}{dS^2} = \frac{d\vec{t}}{dS} = \frac{d\vec{t}}{dt}\cdot\frac{dt}{dS} = \frac{\frac{d\vec{t}}{dt}}{\frac{dS}{dt}} \quad \text{—⑥}$$

$$\vec{n} = \frac{\frac{d\vec{t}}{dt}}{\left|\frac{d\vec{t}}{dt}\right|} \;/\!/\; \frac{d\vec{t}}{dS}$$ (同じ向き)

$\frac{d\vec{t}}{dS}$と同じ向きの単位ベクトル

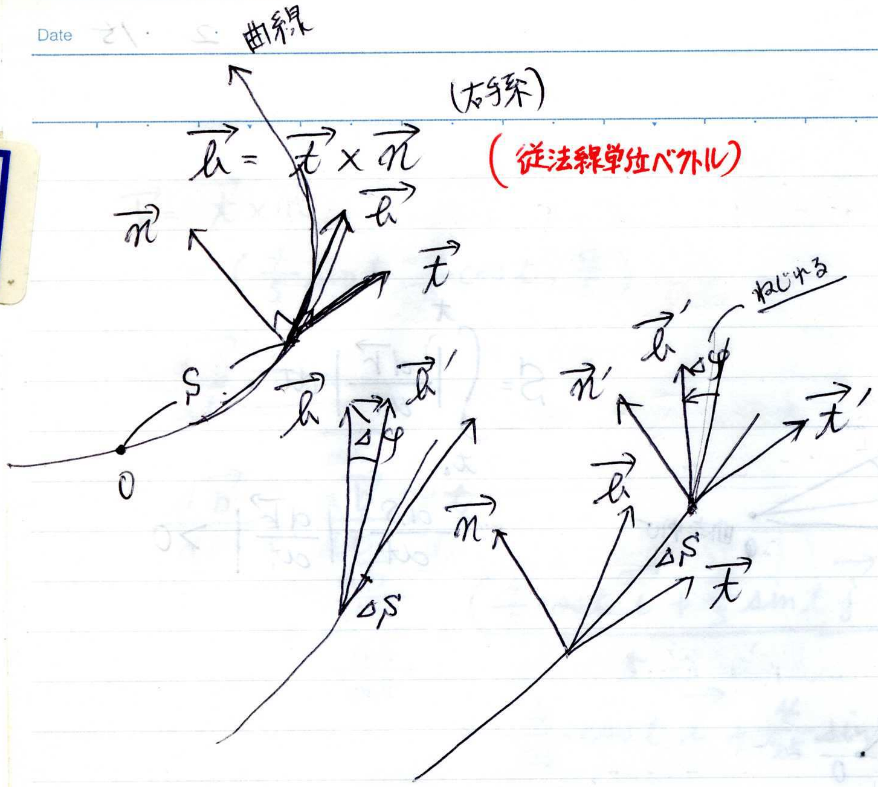
$$\kappa = \left|\frac{d\vec{t}}{dS}\right| = \left|\frac{\Delta\theta}{dS}\right| = \left|\frac{\Delta\theta}{\rho\Delta\theta}\right| = \frac{1}{\rho} \text{ (曲率)} \quad \text{—⑦}$$

ρ(曲率半径)

$$\frac{d\vec{t}}{dS} = \kappa\vec{n} \quad \text{—⑧} \qquad \vec{n} = \frac{1}{\kappa}\frac{d\vec{t}}{dS} \quad \text{—⑨}$$ (主法線単位ベクトル)

(右手系)

$\vec{b} = \vec{t} \times \vec{n}$ (従法線単位ベクトル)



$$0 = \frac{d\vec{b}^2}{ds} = 2\frac{d\vec{b}}{ds}\cdot\vec{b} \quad \text{—①} \qquad \frac{d\vec{b}}{ds} \perp \vec{b} \quad \text{—(ア)}$$

$\vec{t} \perp \vec{b}$ なので

$$\vec{t}\cdot\vec{b} = 0 \quad \text{—②}$$

Sで微分すると $\dfrac{d\vec{t}}{ds}\cdot\vec{b} + \vec{t}\cdot\dfrac{d\vec{b}}{ds} = 0$ —③

$$\kappa\vec{n}\cdot\vec{b} + \vec{t}\cdot\frac{d\vec{b}}{ds} = 0 \quad \text{—④}$$

$\vec{n} \perp \vec{b}$ なので $\vec{n}\cdot\vec{b} = 0$ —⑤

$$\therefore\ \vec{t}\cdot\frac{d\vec{b}}{ds} = 0 \qquad \therefore\ \frac{d\vec{b}}{ds} \perp \vec{t} \quad \text{—(イ)}$$

(ア)(イ)より $\dfrac{d\vec{b}}{ds}$ は $\vec{n}$ に平行である —⑥

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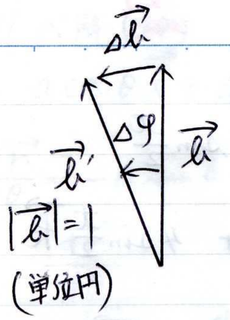

$$\left|\frac{\Delta\varphi}{\Delta s}\right| = |\tau|$$

$$\left|\frac{d\vec{b}}{ds}\right| = \left|\frac{d\varphi}{ds}\right| = |\tau| \quad —⑦$$

とおくと

τ（ねじれ率）（れい率）　（sに対するφの変化率を示す）

$$\therefore \frac{d\vec{b}}{ds} = -\tau\cdot\vec{n} \quad —⑧$$

$$\vec{n} = \vec{b}\times\vec{t} \quad —⑨$$

sで微分する

$$\frac{d\vec{n}}{ds} = \frac{d\vec{b}}{ds}\times\vec{t} + \vec{b}\times\frac{d\vec{t}}{ds} \quad —⑩$$

$$= -\tau\cdot\vec{n}\times\vec{t} + \vec{b}\times\kappa\vec{n} \quad —⑪$$

$$= -\tau\cdot\vec{n}\times\vec{t} + \kappa\,\vec{b}\times\vec{n} \quad —⑫$$

$$\left\{\begin{array}{l}\vec{n}\times\vec{t} = -\vec{b}\\ \vec{b}\times\vec{n} = -\vec{t}\end{array}\right\}$$ を⑫へ代入すると

$$\frac{d\vec{n}}{ds} = -\tau(-\vec{b}) + \kappa\cdot(-\vec{t})$$

$$= \tau\vec{b} - \kappa\vec{t} \quad —⑬$$

$$\boxed{\begin{aligned}\frac{d\vec{t}}{ds} &= \kappa\vec{n}\\ \frac{d\vec{b}}{ds} &= -\tau\vec{n}\\ \frac{d\vec{n}}{ds} &= \tau\vec{b} - \kappa\vec{t}\end{aligned}}$$

フルネー・セネーの公式

[2]

$$x = t - \sin t \qquad y = 1 - \cos t \qquad z = 4\sin\frac{t}{2}$$

$$\vec{r} = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j} + 4\sin\frac{t}{2}\vec{k}$$

$$\frac{d\vec{r}}{dt} = -\cos t\,\vec{i} - \sin t\,\vec{j} + 2\cancel{\sin}\cos\frac{t}{2}\,\vec{k}$$

$$\frac{ds}{dt} = \left|\frac{d\vec{r}}{dt}\right| = \sqrt{\cos^2 t + \sin^2 t + 4\cos^2\frac{t}{2}}$$

$$= \sqrt{1 + 4\cos^2\frac{t}{2}}$$

$$\cancel{\frac{d\vec{r}}{ds} = \frac{d\vec{r}}{dt}}$$

$$\cancel{dt}$$

$$\vec{t} = \frac{-\cos t\,\vec{i} - \sin t\,\vec{j} + 2\cos\frac{t}{2}\vec{k}}{\sqrt{1 + 4\cos^2\frac{t}{2}}}$$

$$\cancel{dt} \qquad \frac{d\vec{t}}{ds} = \frac{d\vec{t}}{dt}\cdot\frac{dt}{ds}$$

$$\frac{d\vec{t}}{ds} = \frac{\frac{d\vec{t}}{dt}}{\frac{ds}{dt}}$$

$\frac{t}{c} + \frac{t}{2}$

$$\cos t = \cos^2\frac{t}{2} - \sin^2\frac{t}{2}$$

$$= \cos^2\frac{t}{2} - \left(1 - \cos^2\frac{t}{2}\right)$$

$$= 2\cos^2\frac{t}{2} - 1$$

$$\cos t + 1 = 2\cos^2\frac{t}{2}$$

$$4\cos^2\frac{t}{2} = 2 + 2\cos t$$

$$\vec{b} = -\sin\theta\sin\alpha\,\vec{i} + \cos\theta\sin\alpha\,\vec{j} - \cos\alpha\,\vec{k}$$

問9 P51 べ解 (安)

$$\vec{r} = a\cos\theta\,\vec{i} + a\sin\theta\,\vec{j} + a\theta\tan\alpha\cdot\vec{k}$$

$$\frac{d\vec{r}}{d\theta} = -a\sin\theta\,\vec{i} + a\cos\theta\,\vec{j} + a\tan\alpha\,\vec{k}$$

(τとKを求めよ)

$$\vec{t} = \frac{\frac{d\vec{r}}{d\theta}}{\left|\frac{d\vec{r}}{d\theta}\right|} \qquad \frac{ds}{d\theta} = \left|\frac{d\vec{r}}{d\theta}\right| = \sqrt{a^2\sin^2\theta + a^2\cos^2\theta + a^2\tan^2\alpha}$$

$$= \sqrt{a^2 + a^2\tan^2\alpha}$$

$$= \sqrt{a^2(1+\tan^2\alpha)} = a\cdot\frac{1}{\cos\alpha}$$

$$= \frac{a}{\cos\alpha}$$

$yz - zy$

$zx - xz$

yz

$zx \quad xz$

$xy - yx$

$$= \frac{d\vec{t}}{ds} = \frac{d\vec{t}}{d\theta}\cdot\frac{d\theta}{ds}$$

$$\vec{t} = -\sin\theta\cos\alpha\,\vec{i} + \cos\theta\cos\alpha\,\vec{j} + \tan\alpha\cos\alpha\,\vec{k}$$

$$= -\sin\theta\cos\alpha\,\vec{i} + \cos\theta\cos\alpha\,\vec{j} + \sin\alpha\,\vec{k}$$

$$\frac{d\vec{t}}{d\theta} = -\cos\theta\cos\alpha\,\vec{i} - \sin\theta\cos\alpha\,\vec{j}$$

$$\frac{d\vec{t}}{ds} = \frac{\frac{d\vec{t}}{d\theta}}{\frac{ds}{d\theta}} = \frac{-\cos\theta\cos\alpha\,\vec{i} - \sin\theta\cos\alpha\,\vec{j}}{\frac{a}{\cos\alpha}} = -\frac{\cos^2\alpha}{a}\left(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}\right)$$

$$\boxed{K = \left|\frac{d\vec{t}}{ds}\right| = \frac{\cos^2\alpha}{a}} \qquad \rho = \frac{1}{K} = \frac{a}{\cos^2\alpha}$$

$$\vec{n} = \frac{\frac{d\vec{t}}{ds}}{K} = \frac{a}{\cos^2\alpha}\left(-\frac{\cos^2\alpha}{a}\left(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}\right)\right)$$

$$\vec{n} = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$$

$$\vec{b} = \vec{t}\times\vec{n} = \left(-\sin\theta\cos\alpha\,\vec{i} + \cos\theta\cos\alpha\,\vec{j} + \sin\alpha\,\vec{k}\right)\times\left(\cos\theta\,\vec{i} + \sin\theta\,\vec{j}\right)$$

$$= \left(-\sin\alpha\,\overset{\sin}{\cancel{\cos}}\,\theta\right)\vec{i} + \left(\sin\alpha\cos\theta\right)\vec{j} + 0\vec{k}$$

$$\left(-\sin^2\theta\cos\alpha - \cos^2\theta\cos\alpha\right)\vec{k}$$